

To: Craig Colby
Noble Denton and Associates Inc.

From: Jun Ying
U.C. Berkeley

Subj: Project Progress Report

Date: May 22, 1995

Dear Craig:

This is the progress report for MODU simulation project (January - April). If you have any comments, please let me know.

Jun Ying

Table of Contents

1. The Markov Chain Modeling of Storm Tracks	1
1.1 Introduction to Markov Chain.....	1
1.2 State Probabilities.....	2
1.3 Hurricane Tracks modeling in Gulf of Mexico.....	4
1.4 Steady State Probabilities	5
1.5 Hurricane Track Modeling for Entire West Coast	7
2. The Updated MODUSIM.....	8
2.1 Three Major Functions of Updated MODUSIM.....	8
2.2 Modeling of Holding after the Collision	10
3. Mooring Analysis	20
3.1 Extreme Response Analysis.....	20
3.2 Quasi-Static and Dynamic Analysis.....	23
3.3 Mooring Analysis in MODUSIM.....	24
3.4 Water Depth Factor	25

1. The Markov Chain Modeling of Storm Tracks

1.1 Introduction to Markov Chain

The state of a system invariably changes with respect to some parameter, for example, time or space. The transition from one state to another as a function of the parameter, or its corresponding transition probability, may generally depend on the prior states. However, if the transition probability depends only on the current state, the process of change may be modeled with the Markov process. If the state space is a countable or finite set, the process is called a Markov Chain. If the transition probability is independent of the state of the system, the process reduces to the Poisson process.

Consider a system with m possible states, namely 1, 2, ..., m , and changes in state can occur only at discretized values of the parameter; for example, at times t_1, t_2, \dots, t_n . Let X_{n+1} denote the state of the system at t_{n+1} . In general, the probability of a future state of the system may depend on its entire history; that is, its conditional probability is:

$$P(X_{n+1} = i | X_0 = x_0, X_1 = x_1, \dots, X_n = x_n) \quad (1)$$

where $|X_0 = x_0, X_1 = x_1, \dots, X_n = x_n$ represent all previous states of the system. If the future state is governed solely by the present state of the system, that is, the conditional probability, Eq.(1) is

$$P(X_{n+1} = i | X_0 = x_0, X_1 = x_1, \dots, X_n = x_n) = P(X_{n+1} = i | X_n = x_n) \quad (2)$$

then the process is a Markov chain. For a discrete parameter Markov chain, the transitional probability from state i at time t_m to state j at time t_n may be denoted by

$$p_{i,j}(m,n) = p(X_n = j | X_m = i); \quad n > m \quad (3)$$

The Markov chain is homogeneous if $p_{i,j}(m,n)$ depends only on the difference $t_n - t_m$; in this case, we define

$$p_{i,j}(k) = p(X_k = j | X_0 = i) = p(X_{k+s} = j | X_s = i) \quad s \geq 0 \quad (4)$$

as the k-step transition probability function. Physically, this represents the conditional probability that a homogeneous Markov chain will go from state i to state j after k times stages. This probability can be determined from the one-step transition probabilities, namely $p_{i,j}(1)$ or simply $p_{i,j}$, between all pairs of states in the system. These transition probabilities can be summarized in a matrix for a system with m states, called the transition probability matrix

$$\mathbf{P} = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,m} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,m} \\ \vdots & \vdots & & \vdots \\ p_{m,1} & p_{m,2} & \cdots & p_{m,m} \end{bmatrix} \quad (5)$$

As the states of a system are mutually exclusive and collectively exhaustive after each transition, the probabilities in each row add up to 1.0. For a homogeneous discrete Markov chain, the probabilities of the initial states are the only other information needed to define the model behavior at any future time.

1.2 State Probabilities

The probabilities of the respective initial states of a system may be denoted by a row matrix

$$\mathbf{P}(0) = [p_1(0), p_2(0), \dots, p_m(0)] \quad (6)$$

where $p_i(0)$ is the probability that the system is initially at state i. In the special case for which the initial state of the system is known, for example, at state i, then $p_i(0) = 1.0$ and all other elements in the row matrix $\mathbf{P}(0)$ are zero. After one transition, the probability that the system is in state j is given by the theorem of total probability as

$$p_j(1) = P(X_1 = j) = \sum_i P(X_0 = i)P(X_1 = j | X_0 = i) \quad (7)$$

Hence,

$$p_j(1) = \sum_i p_i(0) p_{i,j} \quad (8)$$

In matrix notation, the single stages probabilities become

$$\mathbf{P}(1) = \mathbf{P}(0)\mathbf{P} \quad (9)$$

which is also a row matrix.

Similarly, the probability that the system is in state j after two transitions is given by

$$p_j(2) = \sum_k P(X_1 = k)P(X_2 = j|X_1 = k) = \sum_k p_k(1)p_{k,j} \quad (10)$$

or in matrix notation

$$\mathbf{P}(2) = \mathbf{P}(1)\mathbf{P} = \mathbf{P}(0)\mathbf{P}\mathbf{P} = \mathbf{P}(0)\mathbf{P}^2 \quad (11)$$

Therefore, by induction, it can be shown that the n -stage state probability matrix is given by

$$\mathbf{P}(n) = \mathbf{P}(n-1)\mathbf{P} = \mathbf{P}(n-2)\mathbf{P}\mathbf{P} = \dots = \mathbf{P}(0)\mathbf{P}^n \quad (12)$$

1.3 Hurricane Tracks modeling in Gulf of Mexico

For application to hurricane tracks, the state are defined as the direction of one storm track. And the transition step size is two hours. As shown in Fig.1, if we divide the 0-180 to 3 blocks, then there are 3 possible states (1,2,3) and the transition probability matrix **P** is 3×3 .

- 1--direction 0-75
- 2--direction 75-100
- 3--direction 100-180

And based on the statistic analysis of the storm track history, we also assume that, within each state of direction, the moving direction has a probabilistic distribution. They are uniform distribution in state 2 and triangular distribution in state 1 and 3. The distribution functions are shown in Fig.2.

The transition probabilities are estimated by calculating the times of storm track direction changes from one state to the other based on the data of the hurricane track history from MMS. The Table 1 shows the observed transitions in Gulf of Mexico from 1950 to 1992.

Table 1.1 Observed Transitions in Gulf of Mexico (1950 - 1992)

From\To	1	2	3	Total
1	99	12	25	136
2	10	2	4	16
3	14	4	15	33

The $P_{i,j}$ values are estimated from Table 1 using the formula

$$P_{i,j} = \frac{a_{i,j}}{\sum_j a_{i,j}} \quad (13)$$

Table 1.2. $P_{i,j}$ Values

From\To	1	2	3
1	0.73	0.09	0.18
2	0.63	0.12	0.25
3	0.43	0.12	0.45

The resulting transition probability matrix is

$$P = \begin{bmatrix} 0.73 & 0.09 & 0.18 \\ 0.63 & 0.12 & 0.25 \\ 0.43 & 0.12 & 0.45 \end{bmatrix} \quad (14)$$

1.4 Steady State Probabilities

We note that the state probabilities starting with two different initial states approach one another as the number of transition stages increases. In fact, the state probabilities will converge to a set of steady-state probabilities p^* , which are independent of the initial states. Therefore, at steady-state condition,

$$P(n+1) = P(n) = P^* \quad (15)$$

Hence,

$$P(n+1) = P(n)P \quad (16)$$

$$P^* = P^*P \quad (17)$$

For a Markov chain with m states, this matrix equation represents a set of simultaneous equations as follows:

$$[p_1^* \cdots p_m^*] = [p_1^* \cdots p_m^*] \begin{bmatrix} p_{1,1} & \cdots & p_{1,m} \\ \vdots & \ddots & \vdots \\ p_{m,1} & \cdots & p_{m,m} \end{bmatrix} \quad (18)$$

We can find that Eq.(18) contains one degree of freedom. The required constraint to obtain P^* is

$$p_1^* + p_2^* + \cdots + p_m^* = 1.0 \quad (19)$$

Given a particular state matrix P_i , the probabilities of being in the various possible states after n transitions are found from

$$P_{n+i} = P_i \cdot P^n \quad (20)$$

Using the hurricane route input as $P_1 = [1 \ 0 \ 0]$, after two hours, the probabilities matrix is :

$$\begin{aligned} P_2 &= [0.73 \ 0.09 \ 0.18] \\ P_3 &= [0.667 \ 0.098 \ 0.235] \\ P_4 &= [0.649 \ 0.099 \ 0.250] \\ P_5 &= [0.643 \ 0.101 \ 0.256] \end{aligned}$$

And the P^* matrix is calculated as:

$$P^* = [0.643 \ 0.101 \ 0.256] \quad (21)$$

This implies that the hurricane route change according to state 1 about 64.3%, to state 2 about 10.1% and to state 3 about 25.6%. Also, it can be seen that the future transition probability are not strongly dependent upon the present state matrix. After only four transitions, the state probability matrix coverages to the steady-state probability matrix.

1.5 Hurricane Track Modeling for Entire West Coast

The hurricane route history statistics for entire west coast line is also calculated. The result is presented as follow:

Table 1.3 Observed Transitions in Entire West Coast

From\To	1	2	3	Total
1	725	74	135	934
2	55	22	21	98
3	120	19	93	232

Table 1.4. $P_{i,j}$ Values

From\To	1	2	3
1	0.78	0.08	0.14
2	0.56	0.22	0.22
3	0.52	0.08	0.40

The transition probability matrix is:

$$P = \begin{bmatrix} 0.78 & 0.08 & 0.14 \\ 0.56 & 0.22 & 0.22 \\ 0.52 & 0.08 & 0.40 \end{bmatrix} \quad (22)$$

The P^* matrix is calculated as:

$$P^* = [0.708 \quad 0.093 \quad 0.199] \quad (23)$$

From the comparison with the transition matrix of GOF, it can be found that the two matrix are close to each other. It also can be found that storms tends to move clockwise in the GOM and more straight in the entire west coast.

2. The Updated MODUSIM

2.1 Three Major Functions of Updated MODUSIM

The hurricane is assumed to be a storm traveling along a straight line with a given translation speed and direction during the simulation before. Now the storm tracks are modeled with Markov chain so that more MODU's moving information associated with probability can be given from the updated MODUSIM. The three major probability functions that the updated MODUSIM has are:

1. Given MODU location, MODU type, mooring system and surrounding targets, the probability of mooring system failure per year and probability of collision with surrounding target can be estimated.

The old MODUSIM could give this information also with the assumption of storms traveling along a straight line. The updated MODUSIM could change the storm's traveling direction randomly according to a distribution function, so the storm track is a curve during the simulation which is close to the actual storm track. The simulation result will be much more practical and reasonable.

2. Given MODU location, MODU type, mooring systems, surrounding targets and storm track, straight line or curve, determines whether the mooring system failure or not, and also gives the MODU's moving route. This function is very useful when parametric studies are used to verify the result from the MODUSIM when we know an actual MODU's moving route during a storm.

3. Given MODU location, MODU type, mooring systems, surrounding targets, moving directions of possible storms and distance between the storm center and the MODU, predict the possible storm track after Δt and estimate the probability of mooring system failure with probability of collision with surrounding target during this storm.

For example, the data from hurricane Andrew and Zane Barn were used in the simulation. The simulation was begun when the distance between the hurricane center and the MODU was 132 miles, which in another way, we say 12 hour prediction with 11 knots of storm forward speed. Ten possible storm tracks and possible MODU's moving route are presented in Figure 3.

The distance between the storm center and the MODU was changed to 198 miles and 264 miles, which corresponded to 18 hours or 24 hours prediction. The probability of mooring system failure and probability of collision in different simulation are given in Table 1.

Table 2.1. Probability of Failure with Different Warning Time

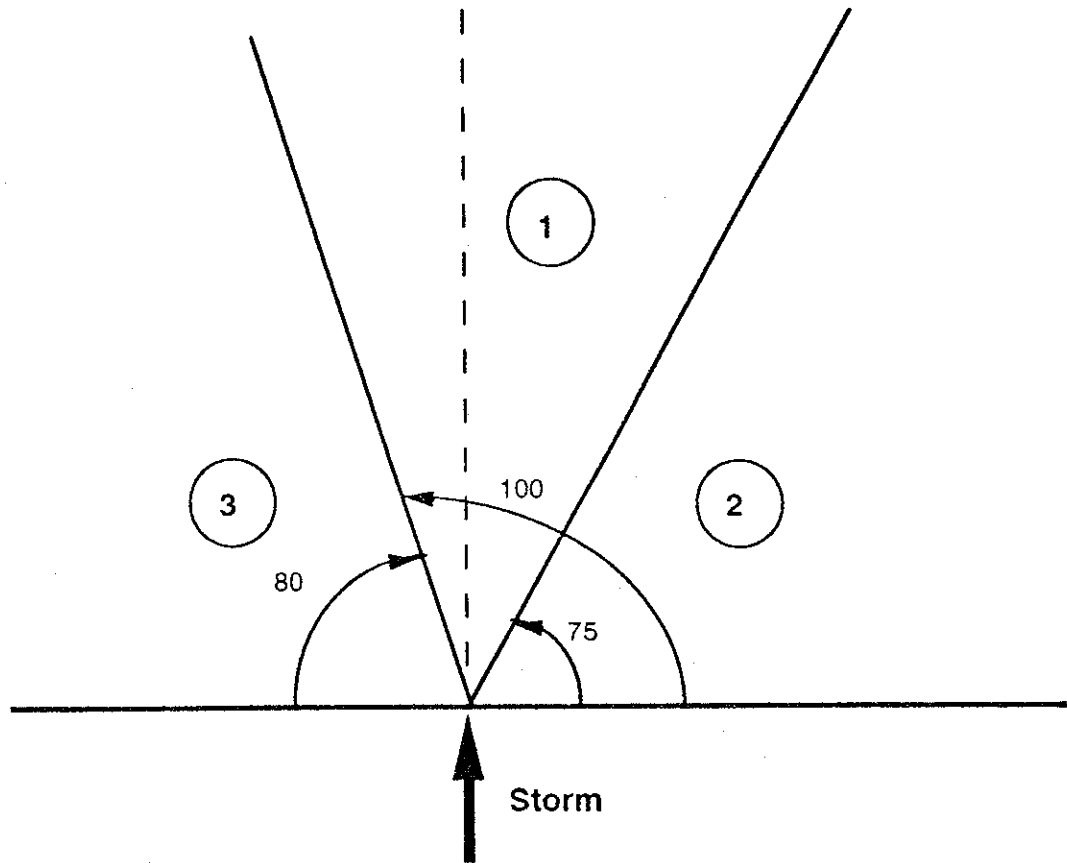
	P% of Mooring Failure	P% of Collision
12 hour	56.5	23.4
18 hour	44.25	19.2
24 hour	30.7	13.6

It is found from the result above that the probability of mooring failure and collision from 12 hours prediction are almost twice of those from 24 hours prediction. These results will be used to develop the early warning system which is proposed to develop next semester.

The directions of the storm track at the beginning of simulations were also changed. It is found that the direction is not an important factor to the probability of collision. This is reasonable since from the transition probability matrix developed before, it is found that after only four transitions, the state probability matrix coverages to the steady-state probability matrix.

2.2 Modeling of Holding after the Collision

During hurricane Andrew, the Zane Barns collided with several platforms after breaking loose from its location. From past experience, we know that the MODU drift direction may have great change after the first collision, or may remain at the collision location for hours before it starts to drift again. The updated MODUSIM can let user decide whether holding happens or not and how long it is hold after collision happens. Figure 4 shows that the MODU's moving route changes a lot if it is held for two hours after the first collision.



**Figure 1. Definition of Storm Track
Transimition State**

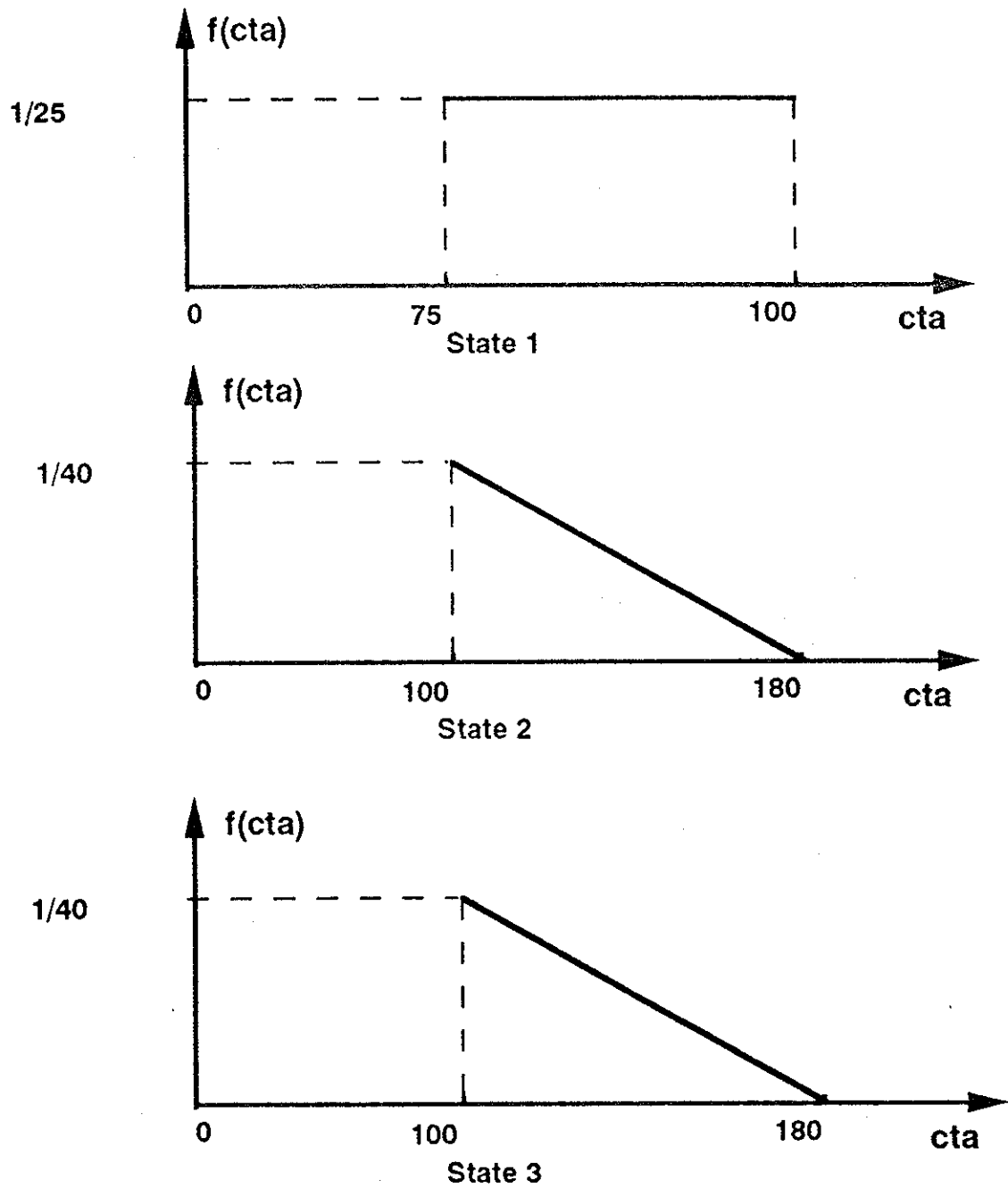
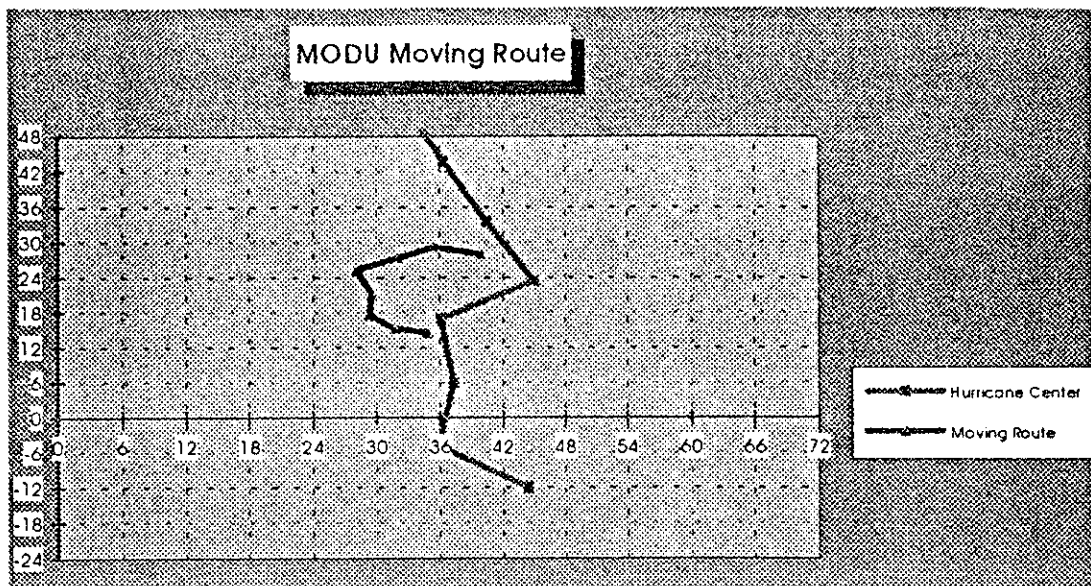
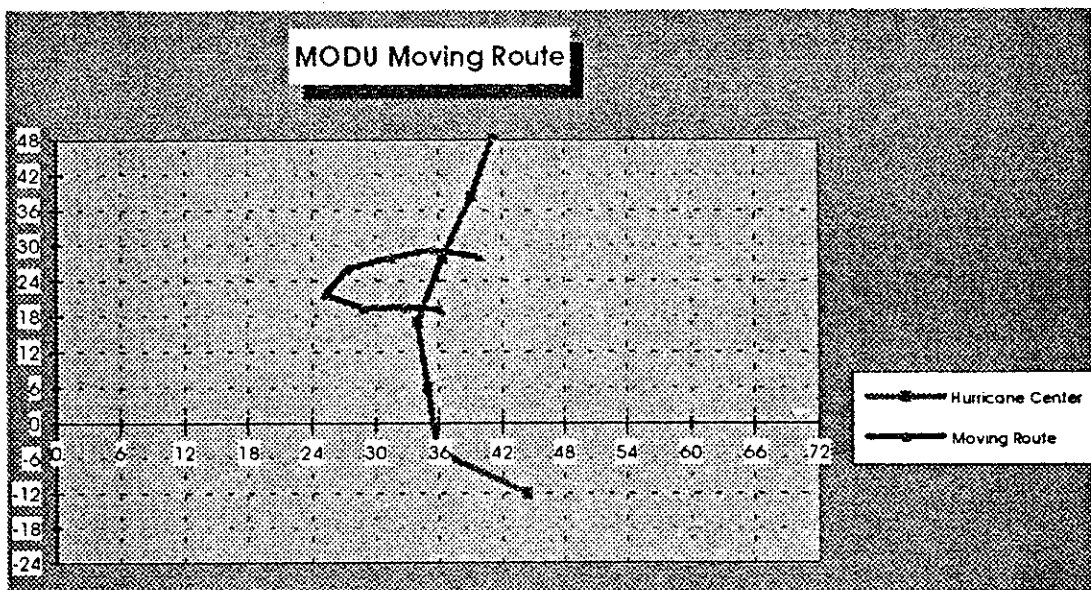


Figure 2. Probability Density Functions of cta in State 1,2 3.

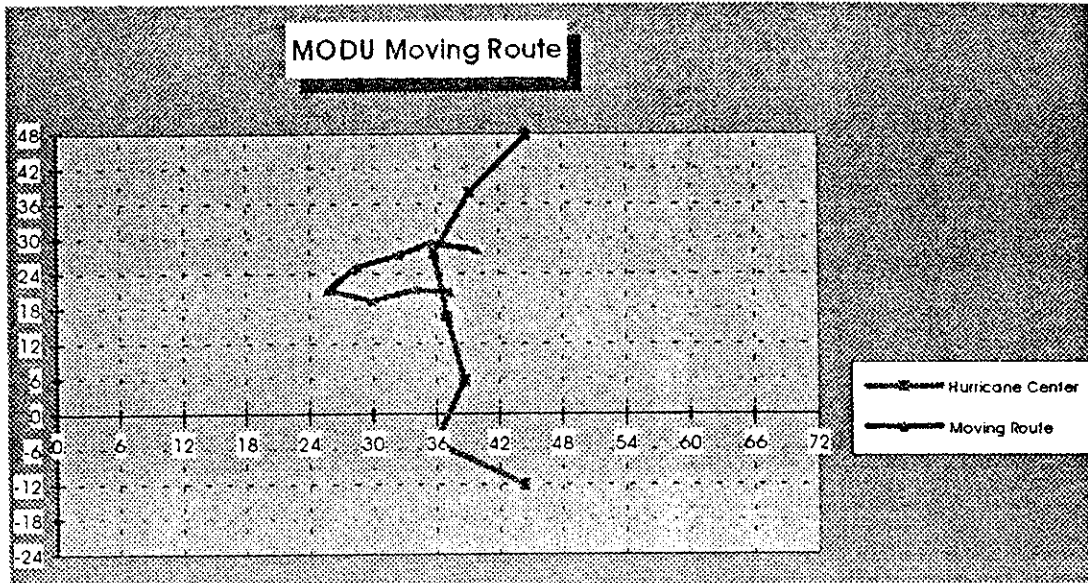


Case 1

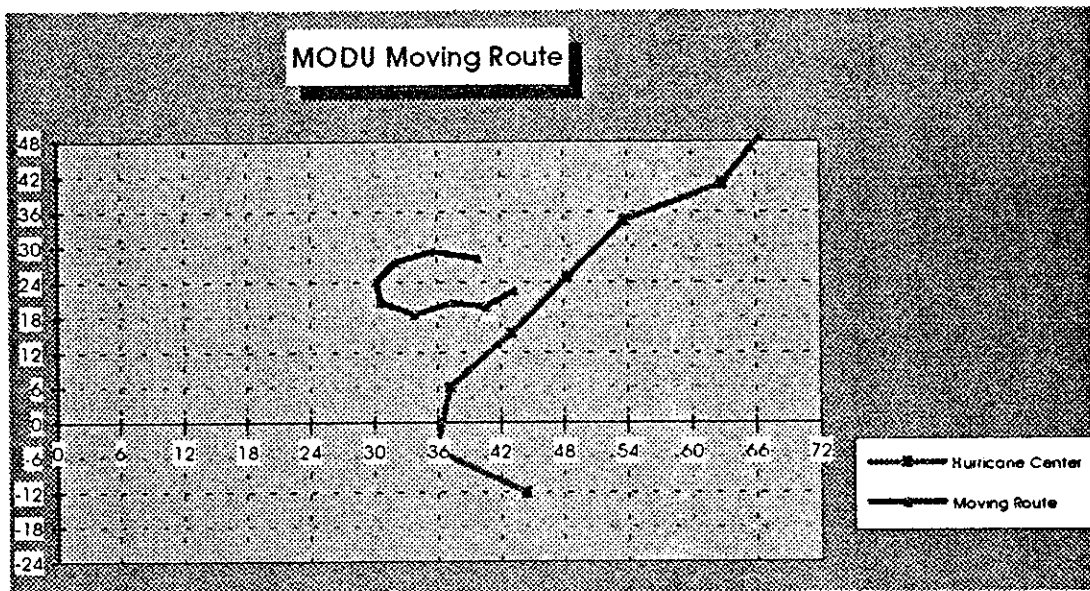


Case 2

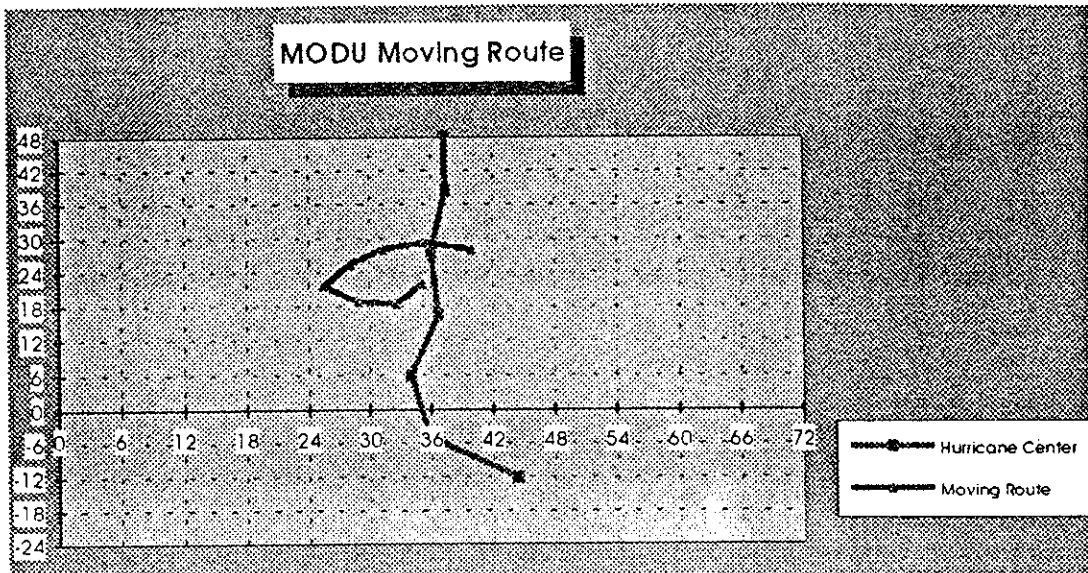
Figure 3



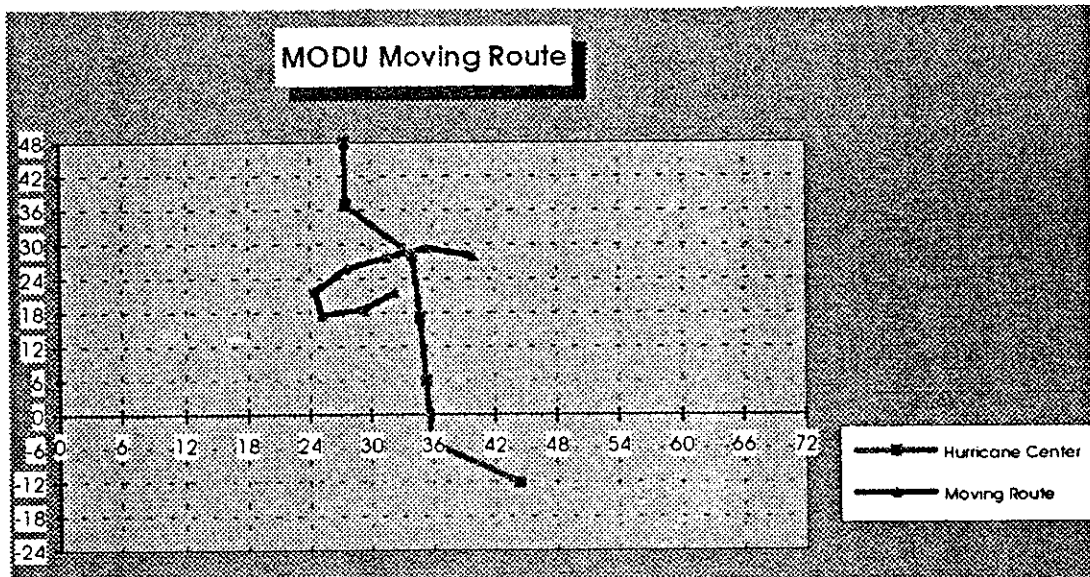
Case 3



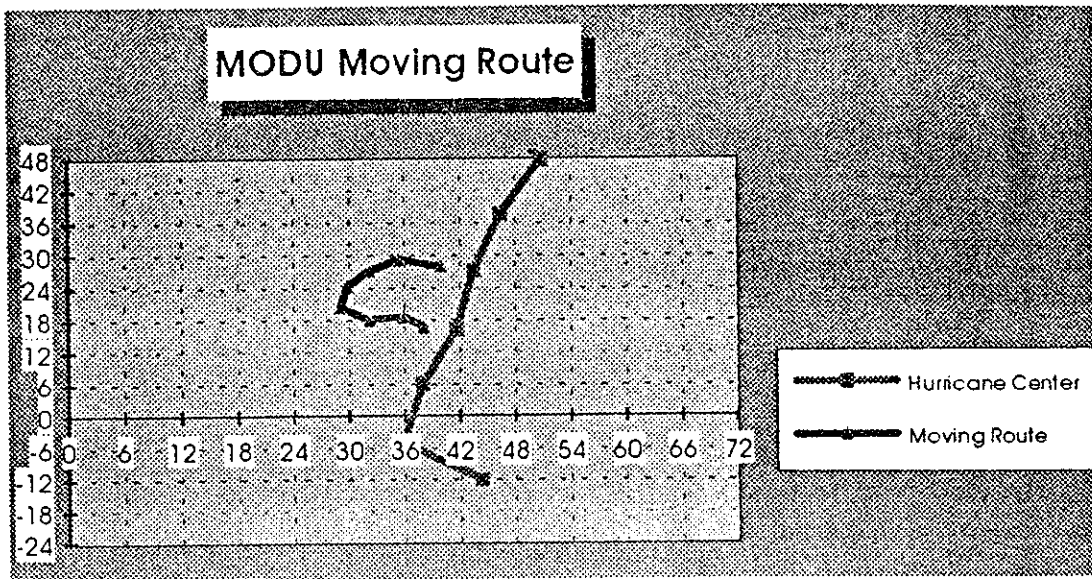
Case 4



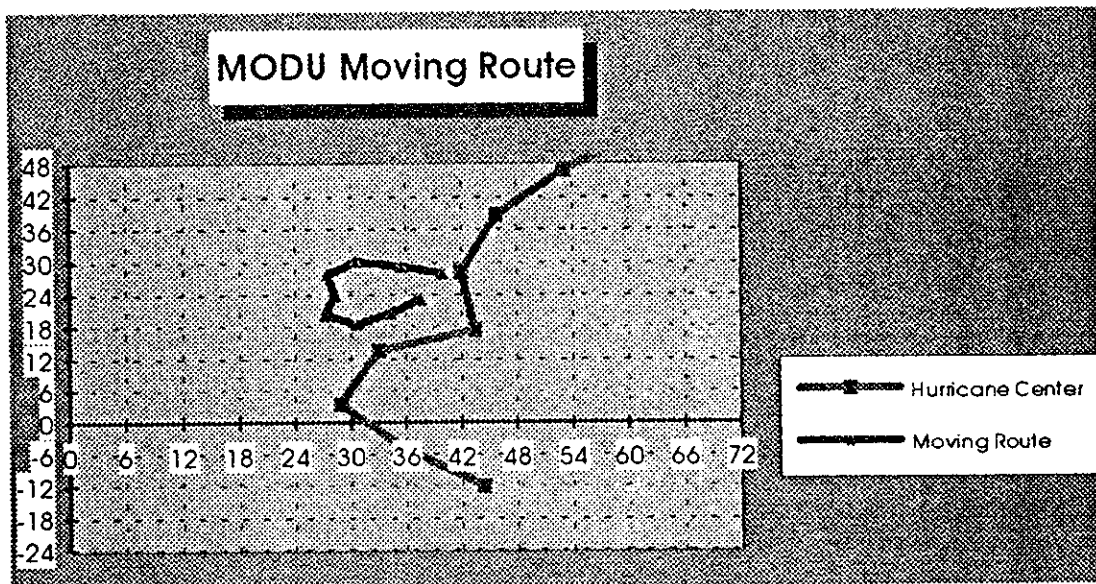
Case 5



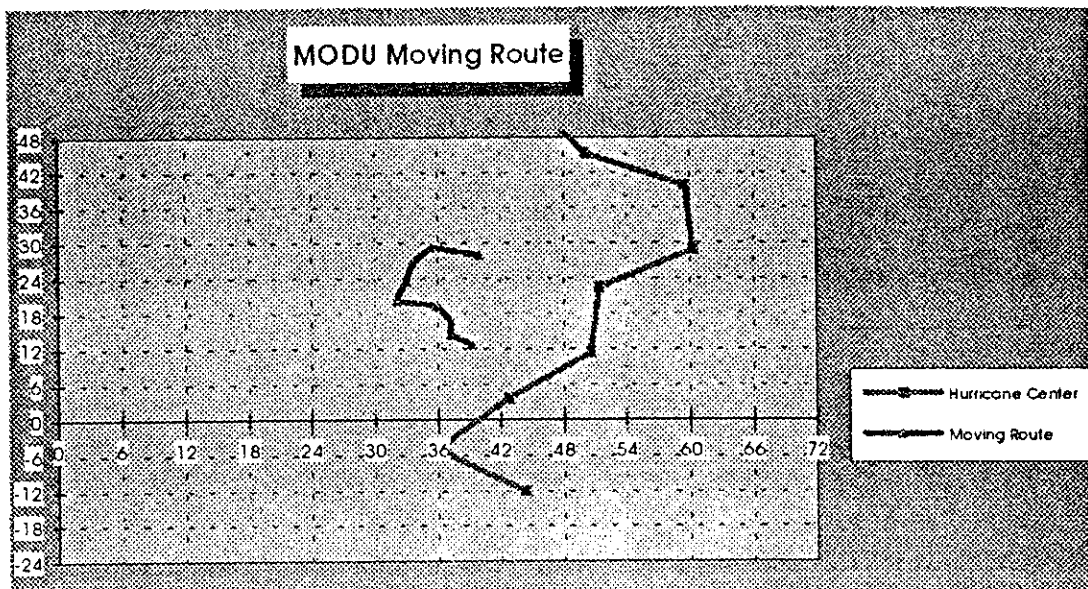
Case 6



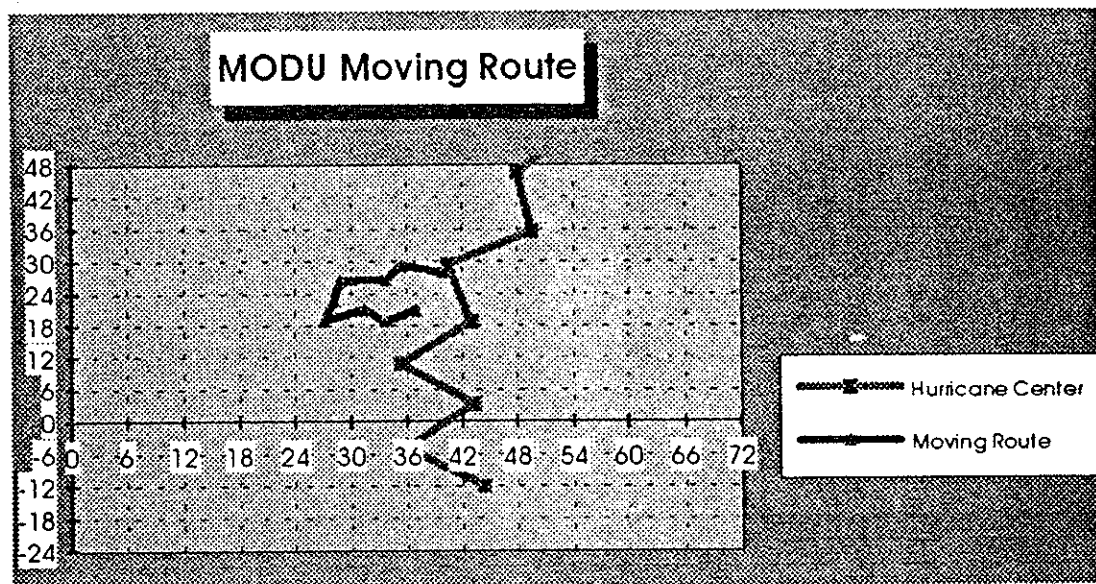
Case 7



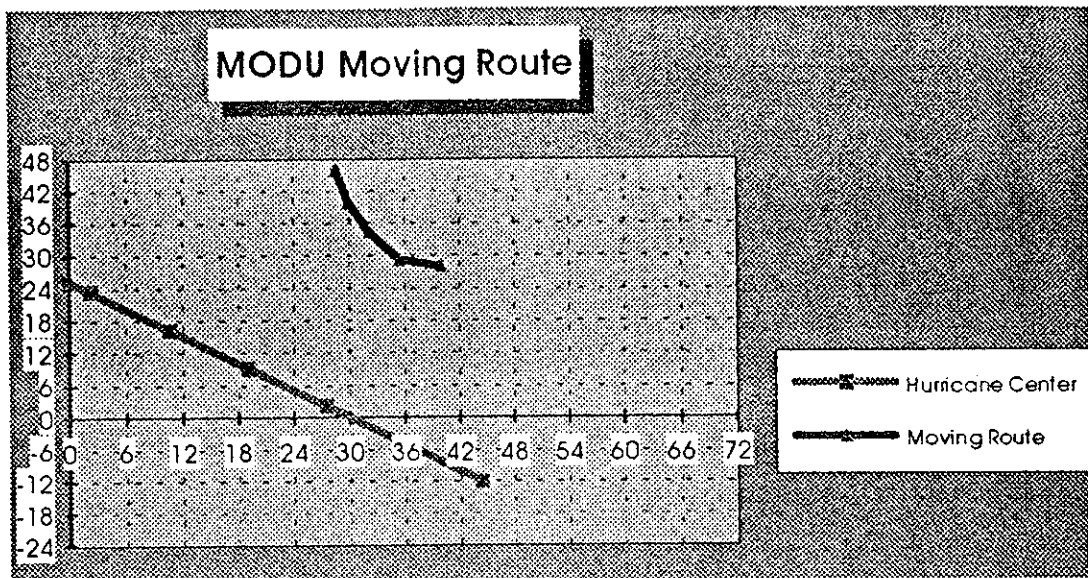
Case 8



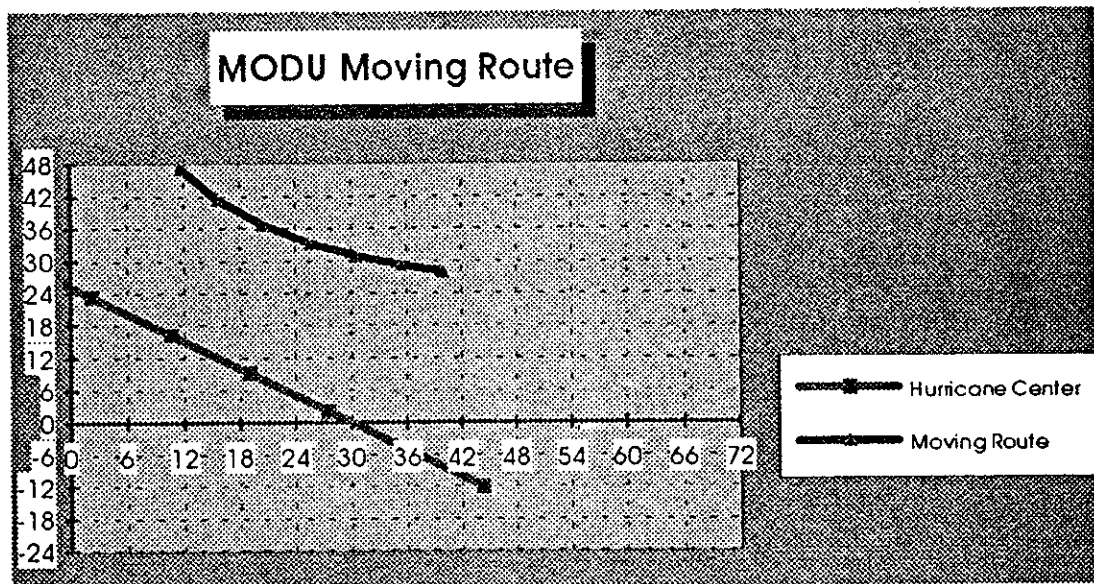
Case 9



Case 10

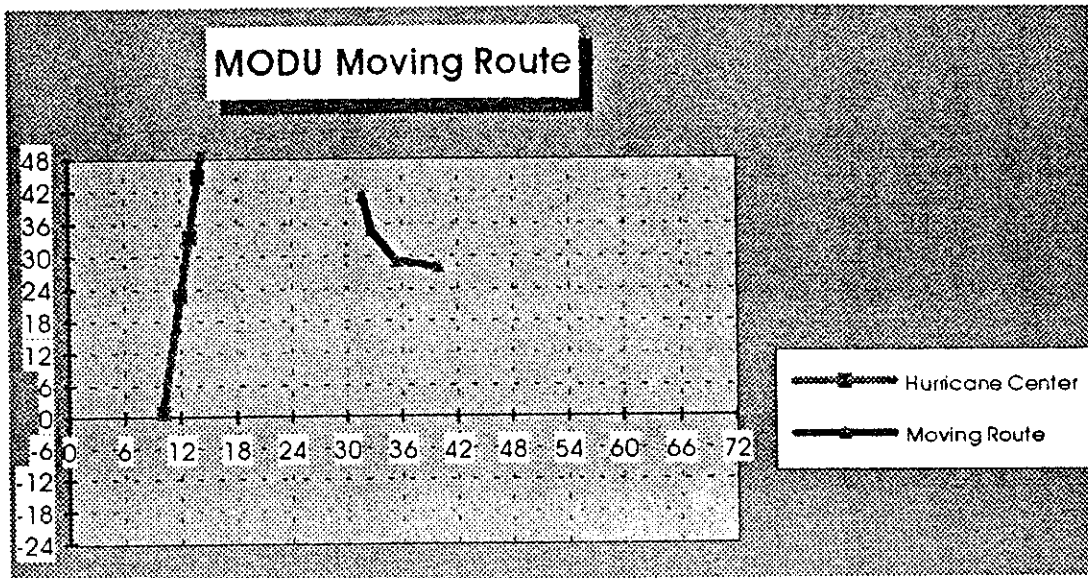


Holding for 2 hours at the First Collision

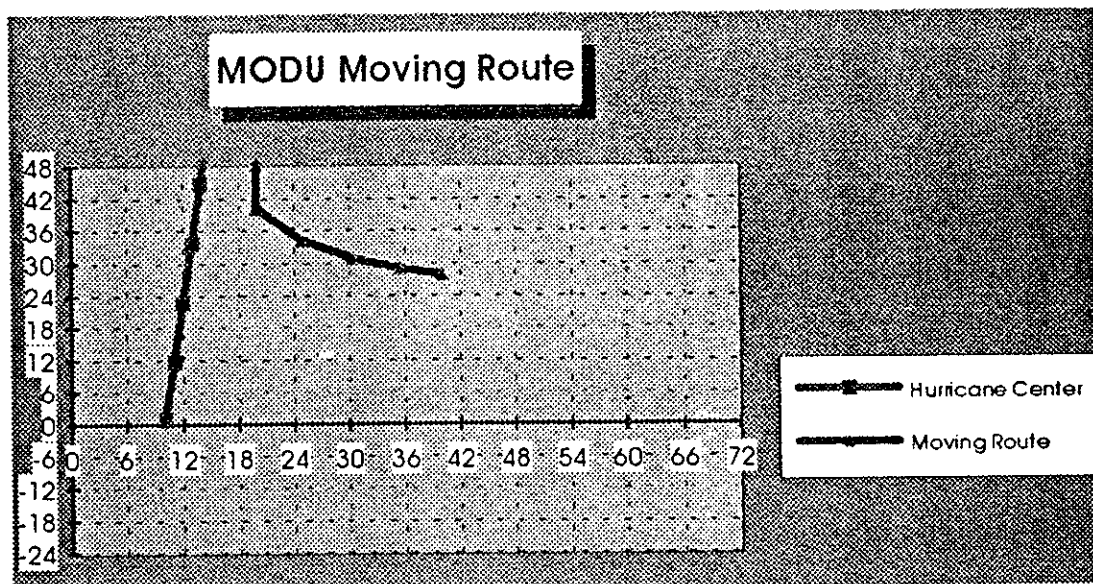


No Holding

Figure 4.



Holding for 2 Hours at First Collision



No Holding

3. Mooring Analysis

3.1 Extreme Response Analysis

Permanent mooring systems should be designed for two primary considerations: system overloading and fatigue. For mobile moorings, only analysis for extreme response is required.

Extreme responses normally govern the design of the FPS mooring. They include vessel offset, mooring line tension, anchor load, and suspended line length. The environmental effects can be divided into three categories:

- Steady state forces including current force, mean wind and mean wave drift forces.
- Low frequency vessel motions due to wind and waves.
- Wave frequency vessel motions.

The responses of a mooring system to mean forces are predicted by static catenary equations. Generally speaking, the responses to low frequency motions can also be predicted by the same method because of the long periods of these motions. The responses to wave frequency vessel motions are usually predicted by one of the following two methods:

(1) Quasi-State Analysis

In this approach, the dynamic wave loads are taken into account by statically offsetting the vessel by an appropriately defined wave included motion. Vessel fairlead motions and dynamic effects associated with mass, damping and fluid acceleration are neglected. Research in mooring line dynamics has shown that the reliability of the mooring designs based on this method can vary widely depending on the vessel type, water depth and line configuration. Therefore, the quasi-static method is not recommended for the final design of a permanent mooring. However, because of its simplicity, this method can be used for

temporary moorings and preliminary studies of permanent moorings with higher factors of safety.

(2) Dynamic Analysis

Dynamic analysis accounts for the time varying effects due to mass, damping, and fluid acceleration. In this approach, the time-varying fairlead motions are calculated from the vessel's surge, sway, heave, pitch, roll and yaw motions. Generally it is sufficient to account for only the vertical and horizontal fairlead motions in the plane of the mooring line. Dynamic models are used to predict mooring line responses to the fairlead motions. Several dynamic analysis techniques are available. The distinguishing feature among various dynamic analysis techniques is the degree to which non-linearity are treated. There are four primary nonlinear effects which can have an important influence on mooring line behavior:

- Nonlinear Stretching Behavior of the Line

The strain or tangential strength of the line is a function of the tension magnitude. Nonlinear behavior of this type typically occurs only in synthetic materials such as nylon. Chain and wire rope can be regarded as linear. In many cases the non linearity can be ignored and a linearized behavior assumed using a representative tangent or secant modulus.

- Changes in Geometry

The geometric non linearity is associated with large changes in shape of the mooring line.

- Fluid Loading

The Merinos equation is most frequently used to represent fluid loading effects on mooring lines. The drag force on the line is proportional to the square of the relative velocity (between the fluid and the line), hence is nonlinear.

- Bottom effects

In most mooring designs, a considerable portion of the line is in contact with the seafloor. The interaction between the line and the seafloor is usually considered to be a frictional process and is hence nonlinear. In addition, the length of grounded line

constantly changes, causing an interaction between this non linearity and the geometric non linearity.

Two methods, frequency domain and time domain analyses, are commonly used for predicting dynamic mooring loads. In the time domain method, all of the nonlinear effects can be modeled. The elastic stretch is mathematically modeled, the full Merinos equation is included, the position of the mooring line is updated at each time step and the bottom interaction is included using a frictional model. The general analysis implies the recalculation of each mass term, dampening term, stiffness term and load at each time step. Hence the computation can become complex and time consuming.

The frequency domain method, on the other hand, is always linear as the linear principle of superposition is used. Hence, all nonlinearities must be eliminated, either by direct linearization or by an iterative linearization.

- Line Stretching

The line stretching relationship must be linearized and a definite value of the modules assumed at each point. The modules can not be a function of line tension but can vary along the line. This is usually not a bad assumption even in the case of synthetic material and, in most cases, a suitable linearization can be achieved.

- Geometry change

In the frequency domain method it is assumed that the dynamic displacements are small perturbations about a static position. The static shape is fixed and all geometric quantities are computed based on this position. The mass, added mass, stiffness, etc. are computed only once. Changes in catenary shape due to the dynamic motion contribution are generally not severe. Hence, a linearization about the position under mean load is generally acceptable.

- Fluid Loads

The nonlinear term in the Merinos equation must be linearized. The quadratic relationship in the relative velocity must be replaced by an equivalent linear relationship. The linearization should take into account the frequency content of the line motion spectrum.

- Bottom Effects

The frictional behavior between the grounded line and the seafloor can not be represented exactly in the frequency domain. Only the "average" or equivalent behavior of the line can be postulated and included. This simplification should be adjusted to the design objective.

3.2 Quasi-Static and Dynamic Analysis

The procedure outlined below is recommended for the analysis of extreme response using a quasi-static or dynamic approach. The calculated response in accordance with this procedure should satisfy the design criteria.

The analysis is normally performed with the following computer programs:

1) Hydrodynamic Motion Analysis programs

These programs are used to determine wave frequency and low frequency vessel motions.

2) Static Mooring Analysis program

This program is used to analyze mooring line response to steady state environmental forces and low frequency motions.

3) Dynamic Mooring Analysis program

This program is used to analyze mooring line response to wave frequency motions.

The recommended analysis procedure is described below:

a) Determine wind and current velocities, and significant wave heights and periods, for both the maximum design, and operating conditions in accordance with guidelines.

b. Determine the mooring pattern, characteristics of chain and wire rope to be deployed, and initial tension.

c. Determine the steady state environmental forces acting on the hull.

- d. Determine the vessel's mean offset due to the steady state environmental forces using the static mooring analysis program.
- e. Determine the low frequency motions. Since calculation of low frequency motions requires the knowledge of the mooring stiffness, the mooring stiffness at the mean offset should be determined first using a static mooring analysis computer program.
- f. Determine the significant and maximum single amplitude wave frequency vessel motions using a hydrodynamic motion analysis program.
- g. Determine the vessel's maximum offset, suspended line length, quasi-static tension, and anchor load.
- h. Determine the maximum line tension and anchor load. A frequency domain or time domain dynamic mooring analysis program should be used.
- i. Compare the maximum vessel offset and suspended line length from step g and maximum line tension and anchor load from step g or h. If the criteria are not met, modify the mooring design and repeat the analysis.

3.3 Mooring Analysis in MODUSIM

Based on the methods above, the mooring capacity of the MODU can be determined. One can get the following formula to determine the mooring capacity of MODU in different environmental condition from the regression analysis:

1. Determine Mean Environmental Force.
2. Determine Mean Offset.

$$\text{Mean.Offset} = A * F_{\text{mean}} + B$$

For example, from the regression analysis of Zane Barn, the equation is:

$$\text{Mean Offset} = 0.019 * F_{\text{mean}} + 0.8842$$

3. Determine Dynamic Offset.

$$\text{Dyn. Offset} = C * H_s^2 + D * H_s + E$$

For Zane Barn:

$$\text{Dyn. Offset} = 0.0073 * H_s^2 + 0.4243 * H_s + 0.1024$$

4. Determine Total Offset.

$$\text{Total. Offset} = \text{Mean. Offset} + \text{Dyn. Offset}$$

5. Determine Maximum Line Tension

$$\text{Tension} = F * \text{Tot. Offset}^2 + G * \text{Tot. Offset} + H$$

For Zane Barn:

$$\text{Tension} = 0.93 * (0.1311 * \text{Tot. Offset}^2 + 17.164 * \text{Tot. Offset} + 200)$$

Parameters A, B, C, D, E, F, G, H are determined by users.

3.4 Water Depth Factor

The algorithm is specific for one rig type at a given water depth and mooring system. The maximum line tension calculated from the processes presented above is a function of vessel type, mooring system, mean offset wave height and water depth. Change any of these makes the constants in the algorithm change. The influence of these different variables can cause havoc when trying to create a simple algorithm. For example, a given dynamic offset may increase tensions significantly in shallow water and have no impact in deep water. With a large mean offset, a small dynamic offset can cause a large increase in tensions. The variables are highly non-linear and difficult to predict without a great deal of research which there is insufficient time to do.

When determining a safe location to stack a MODU, one of the criteria is adequate water depth. To have the best possible chance for survival in a hurricane, this may be interpreted as choosing a location with the optimal water depth for the mooring system. Figures 1 and Figure 2 plot mooring system capacity against water depth for two typical MODUs. The peak of the curves is the optimal water depth for the given mooring system, and as shown in Figures 1 and 2, a water depth between 1000 to 2000 feet generates the sturdiest system.

The mooring system performance curves reflect a series of quasi-state mooring analysis for various water depths using the maximum line length possible. Limitations on line length were as follows: for chain/wire systems, maximum possible while keeping the wire in suspension under pretension loads and for all-chain systems, length was limited by winch capacity in shallow water and maintaining a minimum reserve in deeper water.

These curves show that as a rig is moved into shallow water, the capacity of the mooring system decreases. This reflects a increase in mooring system stiffness as water depth decreases, and for a stiffer system, a given vessel offset will produce larger tensions.

A simple way to solve the current problem is to introduce a "water depth factor" to modify the calculated forces. The forces are calculated as before and multiply them by a "water depth factor", this could then determine an approximate total tension level. This water depth factor can be found from the mooring performance curves mentioned above. See Figure 3.

$$\text{Water Depth Factor} = 0.056 * \left(\frac{\text{Water Depth}}{250} \right)^2 - 0.45 * \left(\frac{\text{Water Depth}}{250} \right) + 1.89$$

This may only be a marginal improvement over the current method but at least it includes the effects of water depth which is the most important factor into the problem.

Table 3.1. Generic Mooring Analysis Data

	Water Depth(ft)	Line Scope(ft)	Mean Forcekips	Mean Offset(ft)	Dyn Offset(ft)	Total Tension
Rig 1	945	5300	269.9	29	6.4	275
Rig 2	1900	7500	647	233	18	560
Rig 3	130	2500	410	3.6	1.8	454
Rig 4	1500	7000	953	47.6	6.9	640
Rig 5	100 m	1200 m	2328 kn.	10.8 m	8 m	3870 kn.

Table 3.2. Mooring Performance vs. Water Depth Data

Water Depth (ft)	Line Scope (ft)	Pretension (kips)	Safety Factor
300	4444	200	1.62
500	4944	200	1.66
750	5644	200	1.71
1000	6144	200	2.08
1500	6811	200	2.20
2000	7644	200	2.28
2500	9044	250	2.22
3000	9500	250	2.24
300	3000	150	2.38
500	3150	150	2.76
750	3350	150	2.96
1000	3475	150	3.08
1500	3600	200	2.85

Figure 1
Mooring System Performance
For Semi-Submersible "A" to 5 yr Storm Criteria

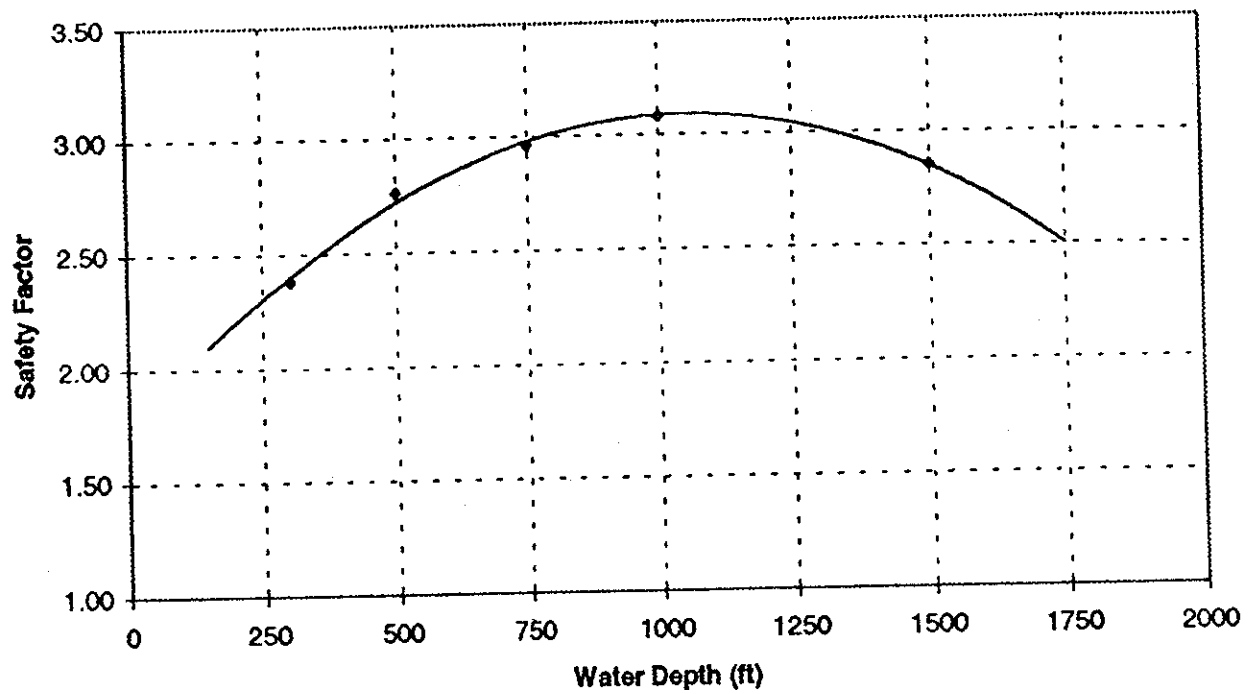
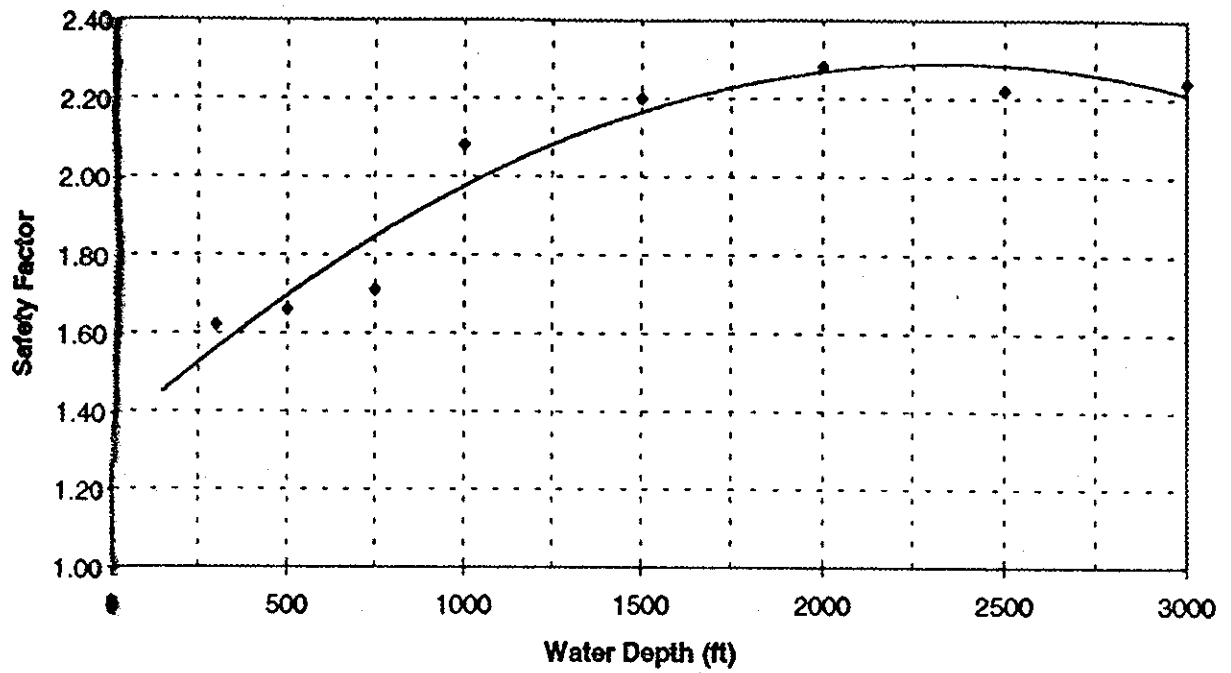


Figure 2
Mooring System Performance
For Semi-Submersible "B" to 5 yr Storm Criteria



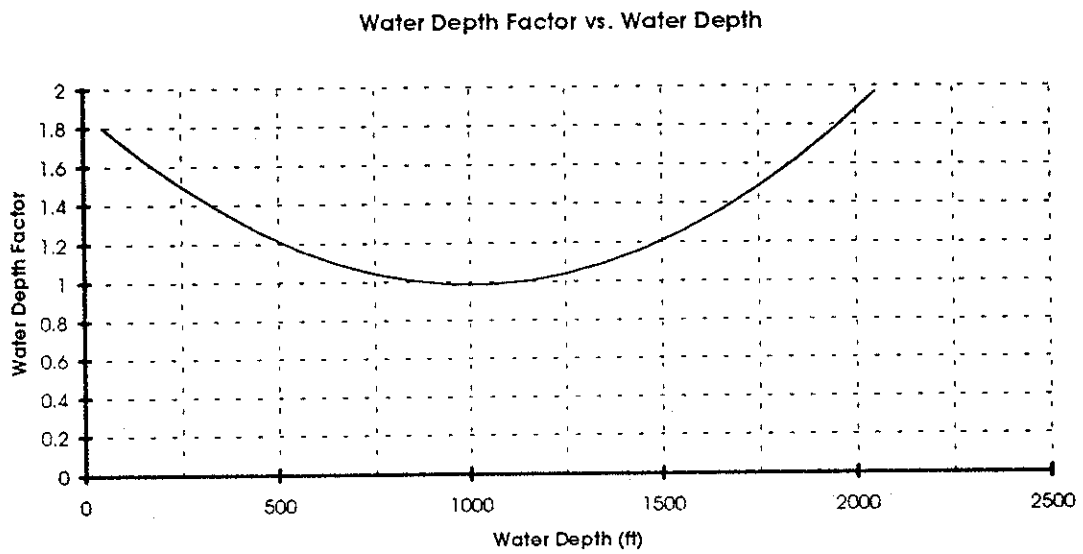


Figure 3.3. Water Depth factor vs. Water Depth